CHES Algorithmics Orientation

5th December 2023

A. Graph Models

B. Graph Models Student Activity

C. Defining Algorithms

D. Defining Algorithms Holiday Homework

2024 Algorithmics Holiday Homework

- **Task 1** River Crossing problem – solution modelled using a **graph** (nodes, edges)
- **Task 2** Egyptian Fractions solved using the following methods
  a) **Exhaustive Search**
  b) **Greedy**

Task 1: River Crossing Problem

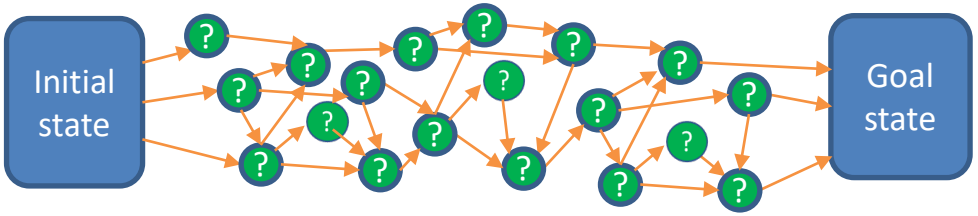
Defining algorithms with Graphs

A farmer returns from the market, where he bought a chicken, a bag of wheat and a dog. On the way home he must cross a river.

- **His boat is very small and won't fit more than one of his purchases.**
- **He cannot leave the chicken alone with the wheat (because the chicken would eat it).**
- **He cannot leave the chicken alone with the dog (because the chicken would be eaten).**

How can the farmer get everything on the other side in this river crossing puzzle?

Model the solution using a graph from initial to goal state.



Task 2:
Egyptian Fractions

Every positive proper fraction can be represented as **sum of unique unit fractions**.

A fraction is a unit fraction if the numerator is 1 and the denominator is a positive integer, for example 1/3 is a unit fraction.

Such a representation of a sum of unique unit fractions is called an Egyptian Fraction as it was used by ancient Egyptians.

Following are few examples:

- Egyptian Fraction Representation of **2/3** is **1/2 + 1/6**
- Egyptian Fraction Representation of **6/14** is **1/3 + 1/11 + 1/231**
- Egyptian Fraction Representation of **12/13** is **1/2 + 1/3 + 1/12 + 1/156**

Exhaustive Search Algorithm for finding Egyptian Fractions

For a given number of the form ‘X/Y’ where Y > X, first find the greatest possible unit fraction, then repeat for the remaining part.

Eg: consider 3/7, we first try 1/2 , but 3/7 < 1/2
We then try 1/3, 3/7-1/3 =9/21-7/21=2/21
We then try 1/4, but 2/21 < 1/4
We then try 1/5, but 2/21 < 1/5
……..
……..

Exhaustive Search tries out all the possible options.

Task 2 a:
Exhaustive Search Algorithm for determining Egyptian Fractions.

Describe in English
or
Define in a Flowchart
or
Define in Structured Pseudocode

Exhaustive Search Algorithm for converting a fraction $^{x}/_{y}$ into Egyptian Fractions

Algorithm EgyptianBruteForce (x,y)

// Input x, the integer numerator of the fraction

// Input y, the integer denominator of the fraction

// Assumption x < y

// Output a list of Egyptian Fractions that add to x/y

Exhaustive search tries out all the possible options.

Greedy Algorithm for Egyptian Fractions

For a given number of the form 'X/Y' where Y > X, first find the greatest possible unit fraction, then repeat for the remaining part.

Eg: consider $\frac{3}{7}$, find the nearest unit fraction less than $\frac{3}{7}$, think about how many times 3 goes into 7, then always round up, the ceiling function always rounds up.

ceiling($\frac{7}{3}$)=3, so try 1/3, $\frac{3}{7} - \frac{1}{3} = \frac{9}{21} - \frac{7}{21} = \frac{2}{21}$

ceiling($\frac{21}{2}$)=11, so try 1/11, $\frac{2}{21} - \frac{1}{11} = \frac{22}{231} - \frac{21}{231} = \frac{1}{231}$

Note: ceiling is a function that always rounds up eg. ceiling($\frac{3}{7}$) will return 1.

Greedy tries out the next best possible option.

Task 2 b:
Greedy Algorithm for determining Egyptian Fractions.

Describe in English
or
Define in a Flowchart
or
Define in Structured Pseudocode

A Greedy Algorithm for converting a fraction $^{x}/_{y}$ into Egyptian Fractions

Algorithm EgyptianGreedy (x, y)

// Input x, the integer numerator of the fraction

// Input y, the integer denominator of the fraction

// Assumption x < y

// Output a list of Egyptian Fractions that add to x/y