## The Knight's Tour

Consider the graph $G_{n}$ that corresponds to possible moves of a knight on a $n \times n$ chessboard. The vertices of the graph are at the centre of each square on the chessboard. Two vertices are adjacent when a knight can move from one square to another in a single move. In graph theory, if two vertices are adjacent, there exists an edge connecting them. For example, in a $3 \times 3$ chess board, all the possible moves from vertex 1 can be shown by

| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 5 | 6 |
| 7 | 8 | 9 |



1) Draw the graph for $G_{5}$, Determine the number of vertices and edges of $G_{5}$.

Now consider the case if the knight is to visit each vertex exactly once in one walk.

2) Find the first value of $n$ for which this occurs, draw a diagram of the chess board for this walk. Label the order of the vertices of your walk.

